第四次作业（10.10）

1. 设 $\mathscr{A}:\quad \mathbb{F}^{n\times 1}\to\mathbb{F}^{m\times 1}$ 为集合映射.

则：$\mathscr{A}$ 为线性映射 $\Longleftrightarrow \pi_i\circ\mathscr{A}$ 为线性映射，$\forall 1\le i\le m$

2. 记 $\mathscr{L}(\mathbb{F}^{n\times 1},\mathbb{F}^{m\times 1})$ 为 $\mathbb{F}^{n\times 1}$ 到 $\mathbb{F}^{m\times 1}$ 的线性映射全体.

对 $\forall A\in\mathbb{F}^{m\times n}$，令 $\ell_A:\quad \mathbb{F}^{n\times 1}\to\mathbb{F}^{m\times 1},\quad \ell_A(X)=AX,\quad \forall X\in\mathbb{F}^{n\times 1}.$

(1) 证明 ℓ_A 为线性映射

(2) 记 $\Phi:\quad \mathscr{L}(\mathbb{F}^{n\times 1},\mathbb{F}^{m\times 1})\to\mathbb{F}^{m\times n}$

$$\Phi(\mathscr{A})=(\mathscr{A}e_1,\mathscr{A}e_2,\cdots,\mathscr{A}e_n),\quad \forall\mathscr{A}\in\mathscr{L}(\mathbb{F}^{n\times 1},\mathbb{F}^{m\times 1});$$

$\Psi:\quad \mathbb{F}^{m\times n}\to\mathscr{L}(\mathbb{F}^{n\times 1},\mathbb{F}^{m\times 1})$

$$\Psi(A)=\ell_A,\quad \forall A\in\mathbb{F}^{m\times n}.$$

证明：Φ,Ψ 为互逆映射.

3. $\forall A\in\mathbb{F}^{m\times n},B\in\mathbb{F}^{n\times p},C\in\mathbb{F}^{p\times q}$，证明矩阵乘法满足下列性质：

(1) $\quad (AB)C=A(BC)$

(2) $\quad AI_n=I_mA=A,\quad I_n=diag(1,\cdots,1)$

(3) $\quad (A_1+A_2)B=A_1B+A_2B,\quad A(B_1+B_2)=AB_1+AB_2$

(4) $\quad (\lambda A)B=A(\lambda B)=\lambda(AB)$

4. 讲义习题三　3,5,6